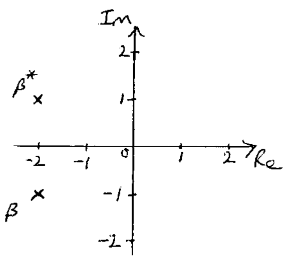
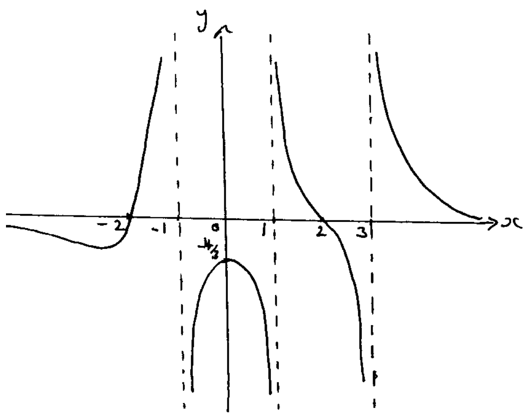
Mark Scheme 4755
June 2007

Section A			
1(i)	$\mathbf{M}^{-1} = \frac{1}{10}\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}$	M1 A1 **[2]**	Attempt to find determinant
1(ii)	20 square units	B1 **[1]**	$2\times$ their determinant
2	$\lvert z-(3-2\text{j})\rvert = 2$	B1 B1 B1 **[3]**	$z \pm (3-2\text{j})$ seen radius = 2 seen Correct use of modulus
3	$x^3-4=(x-1)(Ax^2+Bx+C)+D$ $\Rightarrow x^3-4=Ax^3+(B-A)x^2+(C-B)x-C+D$ $\Rightarrow A=1, B=1, C=1, D=-3$	M1 B1 B1 B1 B1 **[5]**	Attempt at equating coefficients or long division (may be implied) For A = 1 B1 for each of B, C and D
4(i)		B1 B1 **[2]**	One for each correctly shown. s.c. B1 if not labelled correctly but position correct
4(ii)	$\alpha\beta=(1-2\text{j})(-2-\text{j})=-4+3\text{j}$	M1 A1 **[2]**	Attempt to multiply
4(iii)	$\frac{\alpha+\beta}{\beta}=\frac{(\alpha+\beta)\beta^*}{\beta\beta^*}=\frac{\alpha\beta^*+\beta\beta^*}{\beta\beta^*}=\frac{5\text{j}+5}{5}=\text{j}+1$	M1 A1 A1 **[3]**	Appropriate attempt to use conjugate, or other valid method 5 in denominator or correct working consistent with their method All correct

5	**Scheme A**		
	$w = 3x \Rightarrow x = \frac{w}{3}$	B1	Substitution. For substitution $x = 3w$ give B0 but then follow through for a maximum of 3 marks
	$\Rightarrow \left(\frac{w}{3}\right)^3 + 3\left(\frac{w}{3}\right)^2 - 7\left(\frac{w}{3}\right) + 1 = 0$	M1	Substitute into cubic
	$\Rightarrow w^3 + 9w^2 - 63w + 27 = 0$	A3	Correct coefficients consistent with x^3 coefficient, minus 1 each error
	OR	A1 **[6]**	Correct cubic equation c.a.o.
	Scheme B		
	$\alpha + \beta + \gamma = -3$ $\alpha\beta + \alpha\gamma + \beta\gamma = -7$ $\alpha\beta\gamma = -1$	M1	Attempt to find sums and products of roots (at least two of three)
	Let new roots be k, l, m then $k + l + m = 3(\alpha + \beta + \gamma) = -9 = \frac{-B}{A}$ $kl + km + lm = 9(\alpha\beta + \alpha\gamma + \beta\gamma) = -63 = \frac{C}{A}$	M1	Attempt to use sums and products of roots of original equation to find sums and products of roots in related equation
	$klm = 27\alpha\beta\gamma = -27 = \frac{-D}{A}$	A3	Correct coefficients consistent with x^3 coefficient, minus 1 each error
	$\Rightarrow \omega^3 + 9\omega^2 - 63\omega + 27 = 0$	A1 **[6]**	Correct cubic equation c.a.o.
6(i)	$\frac{1}{r+2} - \frac{1}{r+3} = \frac{r+3-(r+2)}{(r+2)(r+3)} = \frac{1}{(r+2)(r+3)}$	M1 A1 **[2]**	Attempt at common denominator
6(ii)	$\sum_{r=1}^{50} \frac{1}{(r+2)(r+3)} = \sum_{r=1}^{50} \left[\frac{1}{r+2} - \frac{1}{r+3}\right]$	M1	Correct use of part (i) (may be implied)
	$= \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) + ...$	M1,	First two terms in full
	$+ \left(\frac{1}{51} - \frac{1}{52}\right) + \left(\frac{1}{52} - \frac{1}{53}\right)$	M1	Last two terms in full (allow in terms of n)
	$= \frac{1}{3} - \frac{1}{53} = \frac{50}{159}$	A1 **[4]**	Give B4 for correct without working Allow 0.314 (3s.f.)

7	$\sum_{r=1}^{n} 3^{r-1} = \frac{3^n - 1}{2}$ $n = 1$, LHS = RHS = 1 Assume true for $n = k$ Next term is 3^k Add to both sides $\text{RHS} = \frac{3^k - 1}{2} + 3^k$ $= \frac{3^k - 1 + 2 \times 3^k}{2}$ $= \frac{3 \times 3^k - 1}{2}$ $= \frac{3^{k+1} - 1}{2}$ But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$. Since it is true for $k = 1$, it is true for $k = 1, 2, 3$ and so true for all positive integers.	B1 E1 M1 A1 E1 E1 **[6]**	 Assuming true for k Attempt to add 3^k to RHS c.a.o. with correct simplification Dependent on previous E1 and immediately previous A1 Dependent on B1 and both previous E marks
			Section A Total: 36

Section B			
8(i)	$(2, 0)$, $(-2, 0)$, $\left(0, \frac{-4}{3}\right)$	B1 B1 B1 **[3]**	1 mark for each s.c. B2 for $2, \ -2, \ \frac{-4}{3}$
8(ii)	$x = 3, \ x = -1, \ x = 1, y = 0$	B4 **[4]**	Minus 1 for each error
8(iii)	Large positive *x*, $y \to 0^+$, approach from above (e.g. consider $x = 100$) Large negative *x*, $y \to 0^-$, approach from below (e.g. consider $x = -100$)	B1 B1 M1 **[3]**	Direction of approach must be clear for each B mark Evidence of method required
8(iv)	Curve 4 branches correct Asymptotes correct and labelled Intercepts labelled 	B2 B1 B1 **[4]**	Minus 1 each error, min 0

9(i)	$x = 1 - 2j$	B1 **[1]**	
9(ii)	Complex roots occur in conjugate pairs. A cubic has three roots, so one must be real. Or, valid argument involving graph of a cubic or behaviour for large positive and large negative *x*.	E1 **[1]**	
9(iii)	**Scheme A**		
	$(x-1-2j)(x-1+2j) = x^2 - 2x + 5$	M1 A1 A1(ft)	Attempt to use factor theorem Correct factors Correct quadratic(using their factors)
	$(x-\alpha)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$	M1	Use of factor involving real root
	comparing constant term: $-5\alpha = 15 \Rightarrow \alpha = -3$	M1	Comparing constant term
	So real root is $x = -3$	A1(ft)	From their quadratic
	$(x+3)(x^2-2x+5) = x^3 + Ax^2 + Bx + 15$ $\Rightarrow x^3 + x^2 - x + 15 = x^3 + Ax^2 + Bx + 15$ $\Rightarrow A = 1, B = -1$	M1 M1 A1 **[9]**	Expand LHS Compare coefficients 1 mark for both values
	OR		
	Scheme B		
	Product of roots $= -15$	M1 A1	Attempt to use product of roots Product is –15
	$(1+2j)(1-2j) = 5$	M1 A1	Multiplying complex roots
	$\Rightarrow 5\alpha = -15$	A1	
	$\Rightarrow \alpha = -3$	A1	c.a.o.
	Sum of roots $= -A$ $\Rightarrow -A = 1 + 2j + 1 - 2j - 3 = -1 \Rightarrow A = 1$	M1	Attempt to use sum of roots
	Substitute root $x = -3$ into cubic $(-3)^3 + (-3)^2 - 3B + 15 = 0 \Rightarrow B = -1$	M1	Attempt to substitute, or to use sum
	$A = 1$ and $B = -1$	A1 **[9]**	c.a.o.
	OR		
	Scheme C		
	$\alpha = -3$	6	As scheme A, or other valid method
	$(1+2j)^3 + A(1+2j)^2 + B(1+2j) + 15 = 0$	M1	Attempt to substitute root
	$\Rightarrow A(-3+4j) + B(1+2j) + 4 - 2j = 0$ $\Rightarrow -3A + B + 4 = 0$ and $4A + 2B - 2 = 0$	M1	Attempt to equate real and imaginary parts, or equivalent.
	$\Rightarrow A = 1$ and $B = -1$	A1 **[9]**	c.a.o.

Section B (continued)			
10(i)	$\mathbf{AB} = \begin{pmatrix} 1 & -2 & k \\ 2 & 1 & 2 \\ 3 & 2 & -1 \end{pmatrix} \begin{pmatrix} -5 & -2+2k & -4-k \\ 8 & -1-3k & -2+2k \\ 1 & -8 & 5 \end{pmatrix}$ $= \begin{pmatrix} k-21 & 0 & 0 \\ 0 & k-21 & 0 \\ 0 & 0 & k-21 \end{pmatrix}$	M1	Attempt to multiply matrices (can be implied)
	$n = 21$	A1 **[2]**	
10(ii)	$\mathbf{A}^{-1} = \frac{1}{k-21} \begin{pmatrix} -5 & -2+2k & -4-k \\ 8 & -1-3k & -2+2k \\ 1 & -8 & 5 \end{pmatrix}$	M1 M1 A1	Use of **B** Attempt to use their answer to (i) Correct inverse
	$k \neq 21$	A1 **[4]**	Accept n in place of 21 for full marks
10 (iii)	**Scheme A** $\frac{1}{-20} \begin{pmatrix} -5 & 0 & -5 \\ 8 & -4 & 0 \\ 1 & -8 & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 12 \\ 3 \end{pmatrix} = \frac{1}{-20} \begin{pmatrix} -20 \\ -40 \\ -80 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$	M1 M1	Attempt to use inverse Their inverse with $k = 1$
	$x = 1,\ y = 2,\ z = 4$	A3 **[5]**	One for each correct (ft)
	OR **Scheme B** Attempt to eliminate 2 variables Substitute in their value to attempt to find others $x = 1,\ y = 2,\ z = 4$	M1 M1 A3 **[5]**	s.c. award 2 marks only for $x = 1,\ y = 2,\ z = 4$ with no working.
			Section B Total: 36
			Total: 72